EXPLORING TEACHERS' NOTION OF LOGICAL THINKING

Roland Fray

Department of Mathematics, University of the Western Cape

The aim in this paper is to explain the outcome of teachers' hands-on experience of solving logical puzzles and their opinions on the usefulness of this in the teaching of geometry.

INTRODUCTION

The aim in this article is to share my experiences in teaching a short course in Euclidean Geometry to educators in the Northern Cape province of South Africa. The Science Learning Centre for Africa at the University of the Western Cape (UWC-SLCA) was contracted by the Northern Cape Education Department and funded by the ETDP- SETA to offer the course in Kimberley recently.

At the start of the course teachers were asked individually to share their experiences in teaching geometry with the rest of the group which consisted of educators of mathematics in the Further Education and Training band (FET band) of the National Senior Certificate, that is, grades ten to twelve. It was especially interesting to hear about the different methods that teachers employed to teach the subject. One teacher likened the solving of a geometry rider to the work of a detective trying to solve a crime that was committed. Another teacher thought of the process of solving a geometry problem as unwinding a piece of wool that became entangled by first finding one endpoint and then to continue the unwinding by untying the knots in the string until you arrive at one long string with the endpoint representing the solution.

One teacher used the method of identifying key words in a rider that related to a theorem or axiom. What is popular from what other teachers described is the method of colouring in on the sketch with colour pens.

Some of the comments made by teachers attending the short course about the learners included:

1. Learners can't think logically.
2. Learners switch off when geometry is taught.
3. Learners have a negative attitude towards geometry.
4. Learners are lazy to think.
5. Learners' inability to read geometry problems with understanding was also mentioned. In this regard the question was raised by one teacher whether learners know and understand the terminology used in the statement of the problem. Another teacher remarked that this could also be ascribed to learners' general reading ability.

These comments are perhaps inter-related with the most important one, in my opinion, being learners' inability to think logically. Among the many reasons for this might be that geometry in the FET phase requires fairly sophisticated thinking processes which are lacking in many learners as a result of not being exposed to this much earlier. For example, it is expected of learners to:

1. Reproduce proofs of theorems and their converses which require the ability to write down their arguments in full mathematical sentences with each statement accompanied by a reason.
2. Integrate their knowledge in solving problems, that is, learners must be able to apply many results in a single problem. It goes without saying therefore that learners must know all the definitions of concepts, theorems as well as their converses (referred to as bookwork).
3. Be familiar with the processes of proving and to understand why proofs are required in mathematics. [Proving a theorem in geometry is very much like putting the pieces of a puzzle together. As in the case of a puzzle where the pieces are placed in position one at a time, the details of a proof must be written down step by step. Each step should be justified by a definition, axiom (mathematical principle), a known result (a theorem or lemma) or some algebraic manipulations.]
4. Understand how new theorems can be discovered and to construct proofs independently and thereby have an appreciation of the ever expanding nature of knowledge.

From the above it should be obvious that it is a complex issue to teach learners how to prove mathematical statements in geometry, in particular. This is because of the many facets that one needs to deal with such as, for example, the ability of learners to read what is required in the problem, the conceptual understanding of the learners, the ability of the learners to carry out algebraic manipulations correctly, the ability of the learners to write their answers in a logical sequence that is coherent and clear as well as the ability of the learners to make appropriate constructions when needed.

It was not clear from teachers' explanations what they meant when they say that learners can't think logically. My impression was that most teachers attending the course had an intuitive idea of what they meant. The aim in this paper is to explain the outcome of teachers' hands-on experience of solving logical puzzles and their opinions on the usefulness of this in the teaching of geometry. It is hoped that educators can use this as a tool in their teaching and thereby introducing learners to the thinking processes required in problem-solving in general, but in Euclidean Geometry, in particular.

Logical Puzzles

At the start of the second session teachers were asked to solve the following puzzles taken from the book by Epp,S, Discrete Mathematics with Applications, Second Edition [3].

(a) The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

1. Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.
2. Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.
3. If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.
4. If lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.
5. If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.
6. If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

   Is it possible for the detective to deduce the identity of the murderer from the above facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death.)

(b) In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humour and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (1 – 5 below) and challenged the reader to use them to figure out the location of the treasure.

1. If this house is next to a lake, then the treasure is not in the kitchen.
2. If the tree in the front yard is an elm, then the treasure is in the kitchen.
3. This house is next to a lake.
4. The tree in the front yard is an elm or the treasure is buried under the flagpole.
5. If the tree in the backyard is an oak, then the treasure is in the garage.

   Where is the treasure hidden?

(c) You are about to leave for school in the morning and discover you don't have your glasses. You know the following statements are true:

1. If my glasses are on the kitchen table, then I saw them at breakfast.
2. I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
3. If I was reading the newspaper in the living room, then my glasses are on the coffee table.
4. I did not see my glasses at breakfast.
5. If I was reading my book in bed, then my glasses are on the bedside table.
6. If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Where are the glasses?

Results

Most teachers attending the course who had the correct answer to a puzzle did not provide any logical reasoning which explained how they got to the answer. There were however a few good attempts where there was clear evidence of logical thinking in the arguments. We will analyze some of these in section 5. We summarize the results obtained by educators in the table below.

	PUZZLE 1	PUZZLE 2	PUZZLE 3
Correct Answer	12	21	14
Incorrect Answer	31	22	28

Selected teachers' responses to the question:

What do you think is the relevance, if at all, of solving puzzles to the teaching of geometry?

Educator 1:

- Dit leer die kinders om alle gegewe inligting te lees en te interpreter.
- Om korrekte afleidings te maak.
- Die feite van die valshede te onderskei(contradictions).
- Om gegewe inligting in verband met mekaar te plaas.
- Om onwaarhede te kanseleer.
- Om ekstra inligting wat geen verband hou met die problem te identifiseer en te elimineer.
- Identifiseer die problem
- Om volgende te bepaal.

- Do not jump to conclusion without proof.
- An interesting way of introducing the thinking skills(logical skills) used in geometry.

Educator 2:

- Identify facts.
- Eliminate unwanted information.
- Reasoning abilities are stimulated.
- Interpretation of statements.
- Read with insight.
- Motivate their answers.
- Logical order justify their answers.

Educator 3:

- Leerders word gestimuleer om logies te redeneer, oplossings te vind en geometry solutions vereis problem-oplossing.
- Logical thinking, reasoning and to find solutions to problems.
- They learn to cancel out some information/ rule out certain facts to come to right answer.
- Kyk na belangrike inligting ⟶ probleemoplossing.
- It is very important to read the information given with understanding and then to identify the facts/ keywords that are relevant in order to solve the problem.
- Eliminate unnecessary information- avoid confusion.

Educator 4:

- Assisted with reading with interpretation.
- Help to form conclusions from contradictory statements.
- Help to eliminate irrelevant information.
- Assists to deduce a true statement
- Help to formulate a conclusion by means of elimination.
- Moves from the unknown to the known.
- Filtering process (like a funnel) to get rid of the unnecessary information.

Analysis of teachers' attempts at solving the three puzzles.

From the results in section 3, it is evident that the majority of the teachers could not solve the puzzles. This demonstrated that educators in the course would benefit a lot from learning more about elementary logic with special emphasis on valid and invalid argument forms. We analyse the attempts of four educators at solving the three puzzles. In the following table we give the solutions for each of the four educators accompanied by my comments at each step of the solution indicating the logical rules of inference that are applied. This indicates that, even though there are some gaps in their arguments some educators attending the course applied logical thinking principles in their solutions of the three puzzles.

Table of four educators solutions of the puzzles:

Educator 1:

ANSWER TO PUZZLE 1	COMMENT
From 5, cook not in kitchen.	Give a reason: only one cause of death ∴ The Butler did not kill Lord Hazelton with a fatal dose of strychnine. ∴ Cook not in kitchen [# 3,modus tollens].
∴ Sara not in dining room	#5, modus ponens.
∴ Lazy Hazelton in dining room	From 2, disjunctive syllogism
∴ Chauffeur killed Lord Hazelton	From 2 and 4, modus ponens.
	#6 represents redundant information.

ANSWER TO PUZZLE 2	COMMENT
From 3 House next to lake	
∴ treasure not in kitchen	# 1, modus ponens
Since treasure not in kitchen elm tree statement not valid	Tree in front yard is not an elm #2, modus tollens
From 4 treasure is buried under the flagpole	#4, disjunctive syllogism
	#5 represents redundant information

ANSWER TO PUZZLE 3	COMMENT
From 4 Did not see glasses at bfast.	
∴ glasses not on kitchen table	From #1 and #4, modus tollens
	∴ I was not reading the newspaper in the kitchen [from #6,modus tollens ∴ I was reading the newspaper in the living room [from 2, disjunctive syllogism
∴ Read newspaper in living room can't be in bedroom because was used to read newspaper	Where does this come from?
∴ left it on coffee table	#3, modus ponens
	#5 represents redundant information.

Educator 2:

ANSWER TO PUZZLE 1	COMMENT
If the cook was in the kitchen the murder weapon does not match the one found	Only one cause of death. ∴ The Butler did not kill Lord Hazelton with a fatal dose of strychnine
So the cook was not in the kitchen	From 3, modus tollens
Which means Sara was not in the dining room	From 5, modus ponens
Which means the Lady was there	From 2, disjunctive syllogism
And that means the chauffeur killed the Lord	# 4,modus ponens
	#6 represents redundant information

ANSWER TO PUZZLE 2	COMMENT
The house is next to a lake	From 3
Therefore the treasure is not in the kitchen	#1, modus ponens
Which means that there is not an elm tree in front of the yard.	From 2, modus tollens
Therefore the treasure is buried under the flagpole	From 4, disjunctive syllogism
	#5 represents redundant information.

ANSWER TO PUZZLE 3	COMMENT
I did not see my glasses at breakfast	# 4
Therefore the glasses are not on the kitchen table	# 1, modus tollens
Therefore I was not reading in the kitchen	#6, modus tollens
But in the living room	#2,disjunctive syllogism
Which means that my glasses are on the coffee table	#3, modus ponens
I could not been reading in the bedroom cause I either read in the living room or the kitchen therefore the glasses cannot be on the bedside.	#5 represents redundant information

Educator 3:

ANSWER TO PUZZLE 1	COMMENT
There was only one cause of death so the butler didn't kill Lord Hazelton with a dose of strychnine. If the butler didn't do it the cook was not in the kitchen	# 3,modus tollens
And Sara not in the dining room	#5,modus ponens
If Sara was not in the dining room Lady Hazelton was in the dining room	#2,Disjunctive Syllogism
The chauffeur killed Lord Hazelton	#4, modus ponens
	#6 represents redundant information
ANSWER TO PUZZLE 2	COMMENT
The house is next to the lake	#3
So the treasure is not in the kitchen	#3 and #1 modus ponens
So if treasure is not in kitchen the tree in front of yard is not an elm	#2, modus tollens
So if the tree in front of yard is not an elm the treasure is buried under the flagpole	#4, disjunctive syllogism
	#5 represents redundant information.
ANSWER TO PUZZLE 3	COMMENT
I did not see my glasses at breakfast	#4
So my glasses is not on kitchen table	#1 and #4, modus tollens
So if my glasses is not on kitchen table I did not read the newspaper in the kitchen	#6, modus tollens
If I did not read the newspaper in the kitchen I was reading the Newspaper in the living room	#2,disjunctive syllogism
So if I was reading in the living room my glasses is on the coffee table	#3, modus ponens
	#5 represents redundant information.

Educator 4:

ANSWER TO PUZZLE 1	COMMENT
The chauffeur killed him.	
1) Either Lady Hazelton or Sara was in the dining room at the time of the murder. 2) So if the cook was in the kitchen then he killed him with a fatal dose of strychnine and that was not the cause of death	∴ The butler did not kill Lord Hazelton with a fatal dose of strychnine ∴ The cook was not in the kitchen
So if the cook was not in the kitchen	
Sara was not in the dining room	#5 ,modus ponens
Which leaves Lady Hazelton	#2, disjunctive syllogism
And if she was in the room the chauffeur killed him	#4, modus ponens.
	#6 represents redundant information.

ANSWER TO PUZZLE 2	COMMENT
The treasure is buried under the flagpole	
Statement #3 states that the house is next to the lake	
So it is not in the kitchen	#1,modus ponens
#4 tree=elm= in kitchen which we ruled out with #3	Not #4 but #2
	∴ from 2 using modus tollens tree not an elm
So in order to have 5 true statements we go with the “or” which is under the flagpole.	∴ #4, disjunctive syllogism
	#5 represents redundant information.

ANSWER TO PUZZLE 3	COMMENT
In the living room on the coffee table.	
It's not in the kitchen because #1 states that if it's there then you saw it	
#4 states that you did not see it	∴ glasses not on kitchen table,#1 and # 4, modus tollens
	∴ I was not reading the newspaper in the kitchen #6,modus tollens
Number 2 states that you were reading in the living room or kitchen	
So that only leaves the living room on the coffee table	#3,modus ponens

The analysis of the solutions as was done above, show clearly the logical rules that the four educators apply, perhaps unknowingly or they might have this ability which distinguishes them from the rest of the group. It is possible for all the participants to learn how to solve such puzzles once they've learnt some elementary logic.

CONCLUSION

Very few teachers could answer all three puzzles correctly. Many wrote the correct answer but were unable to provide a logical explanation. Only five out of the 43 teachers who worked on the puzzles showed evidence of logical thinking in their explanations of their answers. From the analysis of four educators' solutions, it is clear from my comments that the educators applied logical rules of inference in their solutions. Some of their arguments had gaps and it is evident that educators were aware of redundant information that only serves the purpose of confusing the person trying to solve the puzzle. Some of the teachers requested more puzzles. In fact one of the educators reported that her daughter said that learners would enjoy solving such puzzles. The educators' remarks on the usefulness of such puzzles in the teaching of geometry were all positive. It would be interesting to learn whether the teachers are using this in their teaching.

It is unfortunate that due to time constraints we could not give the participants a lot more practice on solving such puzzles

REFERENCES

Bittinger, Marvin L.(1982). Logic, Proof, and Sets 2nd Edition (Addison-Wesley Publishing Company)

Caroll,L. Symbolic Logic and the Game of Logic. New York: Dover Publications Inc.

Epp,S.S.(1995). Discrete Mathematics with Applications,2nd Edition. Belmont,California: Thompson Learning.

Solow, Daniel. (2002). How to read and Do Proofs (An introduction to mathematical thought processes) 3rd Edition John Wiley& Sons.